Multi-Criteria Test Case Prioritization Using Fuzzy Analytic Hierarchy Process

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Abstract—One of the key challenges in software testing today is prioritizing and evaluating test cases. The decision of which test cases to design, select and execute first is of great importance, in particular considering that this needs to be done within tight resource constraints on time and budget. In practice, prioritized selection of test cases requires the evaluation of different test case criteria, and therefore, test case prioritization can be formulated as a multi-criteria decision making problem. As the number of decision criteria grows, application of a systematic decision making solution becomes a necessity. In this paper, we propose an approach for prioritized selection of test cases by using the Analytic Hierarchy Process (AHP) technique. To improve the practicality of the approach in real world scenarios, we apply AHP in a fuzzy environment so that criteria values can be specified using fuzzy variables when precise quantified values are not available. One of the advantages of the proposed decision making process is that the defined criteria with the biggest and most critical role in priority ranking of test cases is also identified. We have applied our approach on an example case in which several test cases for testing non-functional requirements in a systems are defined.

Keywords–Software testing, Test case prioritization, MCDM, Fuzzy AHP, NFR, Fault detection.

I. INTRODUCTION

As the role of software systems in our daily life grows, it becomes more and more important to evaluate and guarantee the quality of such products and ensure that they correctly operate and provide their expected functionality. One way toward this goal is testing of the software product before releasing it to the customers. In simple terms, testing basically means execution of the software system and code with controlled input, in order to evaluate its quality and identify potential problems in it. Through testing, we can increase our confidence in the quality of the product.

In this context, it is reasonable to assume that the more tests that are performed out of a diverse and high-quality test suite, the greater should be our confidence in the product quality be. But in practice, there is usually limited resources (in terms of time, budget, personnel, etc.) available and allocated for testing activities.

Among different testing activities, our focus in this paper, is on the prioritization of test cases in a test suite, regardless of how they are created (i.e., manually implemented, automatically generated, or a mixed approach). Considering such resource limitations, it becomes very important that from all possible test cases that can be considered for a system, a good subset, which fits the available resources will be selected. Selection of appropriate test cases from a test suite can be done based on a number of different criteria. Considering that in practice and in industrial settings a test suite can consist of a large number of different test cases, it is necessary to apply a systematic approach for the selection of appropriate test cases (based on the identified criteria) from the set of all test cases existing in the test suite.

Generally, the term test case selection is used to refer to the techniques which aim to reduce the number of test cases that are executed. Test case prioritization techniques, on the other hand, are used to order test cases in a way that the most important ones, i.e., those which can lead to a higher and increased rate of fault detection are run earlier [1]–[3]. Test case prioritization can be particularly necessary in performing regression testing which is also an expensive testing process [2], [4] where (some) test cases are selected and executed several times.

In [5], we have introduced an approach for prioritization of test cases based on the result from model-based trade-off analysis of non-functional requirements. In that approach, by performing analysis on the model of non-functional requirements, parts of the system that can have more severe problems with respect to the satisfaction of such requirements are identified. This is done by calculating a deviation indicator value for non-functional requirements as part of the model analysis.

Then, assuming that a cost-effort value for each test case and a total cost-effort budget to perform testing activities are known, we prioritize test cases that target requirements with higher deviation indicator value (thus potentially more problematic parts of the system) while fitting the total cost-effort budget available (prioritization and selection).

In this paper, the test case prioritization part of our work in [5] is extended to enable prioritization decisions based on multiple criteria. To enable this, we apply the Analytic Hierarchy Process (AHP) technique. The importance of multi-criteria test case prioritization solutions is also recognized and emphasized in the literature, see for example [6].

Since in practice it is generally hard or sometimes impossible to provide precise values for different criteria [7] and properties of a test case such as fault detection probability, we apply AHP in a fuzzy environment so that users can specify criteria values in the form of fuzzy variables (e.g., high, low, etc.) and thus make the overall approach more practical and usable in real scenarios.

The term decision making in a fuzzy environment means a decision making process in which the goals and/or the constraints, but not necessarily the system under control, are fuzzy in nature. This means that the goals and/or the constraints constitute classes of alternatives whose boundaries are not sharply defined [8].

While in this paper a particular set of criteria are selected based on which prioritization decisions are made, the fuzzy AHP approach can be well used for any other set and number of criteria. It should also be noted that in this context, our test case prioritization approach in this paper, prioritizes and orders test cases based on various criteria and not necessarily and merely based on their early rate of fault detection, which provides thus a broader scope of prioritization than what is defined, for instance, in [1], [3] for test case prioritization.

Another important property of the approach is that the most critical decision criterion, i.e., the criterion which has the biggest role in the ranking of test cases, is also identified. The main contributions of the paper are thus the following:

- a novel multi-criteria test case prioritization method based on AHP,

- application of the method in a fuzzy environment to relax the need of having precise values for criteria,

- an illustration of the method using a case study on laptop customization, and

- a brief analysis and discussion of the results.

The remainder of the paper is structured as follows. In Section I-A, the related literature is reviewed. Section II gives the theoretical background for fuzzification and fuzzy multiple-criteria decision-making. In Section III, the proposed approach and suitable test-case properties are described. Section IV gives an example of applying fuzzy AHP to the testing of non-functional properties of a customized laptop computer system. Finally, Section V gives conclusions and recommendation for future research.

A. Related Work

In [3], [4] the initial problem has been assumed as a single criterion decision making problem. In [4] test cases have been prioritized by the rate of fault detection and the authors used a weighted average of the percentage of faults detected, which is not a direct measure of the rate of fault detection. In [3] the authors used a code-coverage based greedy algorithm for prioritizing the test cases. Since single criterion decision making is hard to cover a larger or more complex problem, we investigate multi-criteria prioritization in the present work. Techniques of multi-criteria decision making are in contrast able to cover a large number of criteria for several test cases. We further define the initial problem in a fuzzy environment by using the linguistic variables. The effect of the criteria on the test cases have been interpreted as a fuzzy set which allows the expression of imprecise properties.

II. BACKGROUND & PRELIMINARIES

Testing an embedded system is a costly activity in terms of time and budget consumption. Considering these two limiting factors in testing of a system and the aim of companies to reduce time-to-market for their products, only a certain number of test cases can be selected to execute. In this section, we introduce AHP as a suitable decision making technique and redefine this method in a fuzzy environment. The AHP applies a Decision Support System (DSS) model in selection of alternatives. DSS is a computer based information system which supports data mining, decision modelling and prioritization to solve structured and unstructured problems [9].

A. Fuzzification

Fuzzy truth represents membership in vaguely defined sets, and variables over these sets are called fuzzy variables. From a user perspective, fuzzy properties are often described using linguistic variables. This section outlines the process of transforming a linguistic value into a fuzzy value. For a full introduction to fuzzy mathematics [10].

Definition 1: A linguistic variable indicates a variable whose values are words or sentences in a natural or artificial language [11].

As an example, a fuzzy property such as *age* could be described using the values "young", "fairly old", and "middle-aged".

Fuzzification consists of the process of transforming the linguistic variables to fuzzy sets [12]. We use grade of membership to associate a value, indicating the degree of truth, to each linguistic term. Some basic concepts of fuzzy logic which are relevant for this work is based on the definitions provided by Zadeh [13], Yun Shi [12], Yang [14] and Kerre [15], and are revisited here briefly.

Definition 2: A fuzzy set is a pair (A, m_A) where A is a set and $m_A : A \to [0, 1]$; for each $x \in A$, $m_A(x)$ is called the grade of membership of x in (A, m_A).

The grades of membership of 0 and 1 correspond to the two possibilities of truth and false in an ordinary set [13].

Let $x \in A$; then x is called fully included in the fuzzy set (A, m_A) if $m_A(x) = 1$ and is called not included if $m_A(x) = 0$. The set $\{x \in A \mid m_A(x) > 0\}$ is called the support of (A, m_A) and the set is called a kernel, where x is a fuzzy member if $0 < m_A(x) < 1$ (see [14]).

One-dimensional membership functions have different shapes such as triangular, trapezoidal or Gaussian shape. In this paper, we use five triangular-shaped membership functions, illustrated in Figure 1.
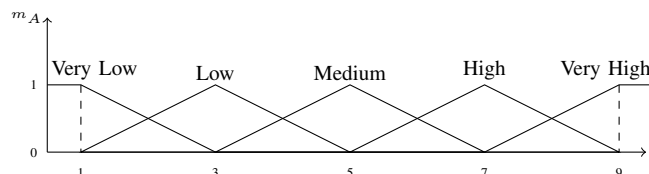

Figure 1. The five fuzzy membership functions for the linguistic variables

Definition 3: A triangular fuzzy number (TFN) can be defined as a triplet $M = (l, m, u)$ where l, m, u are real numbers and l indicates low bound, m is modal and u represents a high bound [16].

B. Fuzzy Multiple Criteria Decision Making

Multi-criteria decision making (MCDM) for structuring decision problems and evaluating alternatives provides a rich collection of methods [17]. The decision maker's role in the decision making situations is to evaluate the effect of different criteria on the existing alternatives to choose the best one among them. In addition, the decision makers will need to choose an appropriate technique for decision making, which depends on the problem statement, limitation and constraints.

Since Zadeh and Bellman and a few years later Zimmermann developed the theory of decision support systems in a fuzzy environment, different techniques such as TOPSIS

(The Technique for Order of Preference by Similarity to Ideal Solution), QSPM (Quantitative Strategic Planning Matrix), AHP and etc., have been developed for solving various multi-criteria decision making problems [18].

In the present work, we use the fuzzy analytic hierarchy process (FAHP), for solving our problem. The AHP approach is an example of a heuristic algorithm, which based on the comparison matrix with triangular fuzzy numbers (TFN). Here, we provide a summary of how AHP is extended in a fuzzy environment.

As some other decision making techniques, AHP has also some weak points. One of the disadvantages of AHP is possible disagreement between decision makers. If, for example, more than one decision maker is working on the decision support system, different viewpoints about the linguistic variables of each criterion can complicate matters [19]. Therefore, using a TFN instead of a constant has been suggested as a good solution. By using Table I, the decision makers are able to interpret the linguistic variables in the form of TFNs.

TABLE I. THE FUZZY SCALE OF IMPORTANCE

Fuzzy number	Description	Triangular fuzzy scale	Domain	$m_A(x)$
$\tilde{9}$	Very High	$(7, 9, 9)$	$7 \leq x \leq 9$	$(x-7)/(9-7)$
$\tilde{7}$	High	$(5, 7, 9)$	$7 \leq x \leq 9$	$(9-x)/(9-7)$
			$5 \leq x \leq 7$	$(x-5)/(7-5)$
$\tilde{5}$	Medium	$(3, 5, 7)$	$5 \leq x \leq 7$	$(7-x)/(7-5)$
			$3 \leq x \leq 5$	$(x-3)/(5-3)$
$\tilde{3}$	Low	$(1, 3, 5)$	$3 \leq x \leq 5$	$(5-x)/(5-3)$
			$1 \leq x \leq 3$	$(x-1)/(3-1)$
$\tilde{1}$	Very Low	$(1, 1, 3)$	$1 \leq x \leq 3$	$(3-x)/(3-1)$

As Table I represents, every linguistic variable has been defined by a TFN. In some disagreement situations, the geometric mean of the TFNs can be used as a final agreement. As mentioned earlier, AHP is based on a series of pairwise comparisons of alternatives and criteria.

In a fuzzy environment, the linguistic variables that we have defined in Table I based on the standard 9-unit scale [16], are used to make the pairwise comparisons. The fuzzy comparison matrix $A = (\tilde{a}_{ij})_{n \times n}$ can be formulated and structured as [20]:

$$A = \begin{pmatrix} (111) & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & (111) & \dots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \dots & (111) \end{pmatrix} \quad (1)$$

where $\tilde{a}_{ij}$ $(i = 1, 2, ..., n, j = 1, 2, ..., m)$ is an element of the comparison matrix and the reciprocal property of the comparison matrix is defined as $\tilde{a}_{ij} = \tilde{a}_{ij}^{-1}$. The pairwise comparisons need to be applied on every criteria and alternatives, and the values for $\tilde{a}_{ij}$ come from a predefined set of fuzzy scale value as showed in Table I. Then $\tilde{a}_{ij}$ represents, a TFN in the form of $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$ and matrix A consists of the following fuzzy numbers:

$$\tilde{a}_{ij} = \begin{cases} 1 & i = j \\ \tilde{1}, \tilde{3}, \tilde{5}, \tilde{7}, \tilde{9} \quad or \quad \tilde{1}^{-1}, \tilde{3}^{-1}, \tilde{5}^{-1}, \tilde{7}^{-1}, \tilde{9}^{-1} & i \neq j \end{cases}$$

After we create the comparison matrix A, we need to find a priority vector of matrix A.

To make it, we need to calculate the value of fuzzy synthetic

extent $\tilde{S}_i$ for each row in matrix A by [16]:

$$\tilde{S}_i = \sum_{j=1}^{m} \tilde{a}_{ij} \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} \tilde{a}_{ij} \right]^{-1} \quad (2)$$

where $\tilde{a}_{ij}$ is a TFN, $\otimes$ is the fuzzy multiplication operator and:

$$\sum_{j=1}^{m} \tilde{a}_{ij} = \left(\sum_{j=1}^{m} l_{ij}, \sum_{j=1}^{m} m_{ij}, \sum_{j=1}^{m} u_{ij} \right), \forall i = 1, 2, ..., n, \quad (3)$$

also

$$\left[\sum_{i=1}^{n} \sum_{j=1}^{m} \tilde{a}_{ij} \right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} \sum_{j=1}^{m} u_{ij}}, \frac{1}{\sum_{i=1}^{n} \sum_{j=1}^{m} m_{ij}}, \frac{1}{\sum_{i=1}^{n} \sum_{j=1}^{m} l_{ij}} \right) \quad (4)$$

now we can compute the degree of possibility for the TFNs.

Definition 4: Let $\tilde{a}_1 = (l_1, m_1, u_1)$ and $\tilde{a}_2 = (l_2, m_2, u_2)$ be two TFNs, the degree of possibility of $\tilde{a}_1$ to $\tilde{a}_2$, $V(\tilde{a}_2 \geq \tilde{a}_1)$, can be obtained as [16]:

$$V(\tilde{a}_2 \geq \tilde{a}_1) = \begin{cases} 1 & \text{if } m_2 \geq m_1, \\ 0 & \text{if } l_1 \geq u_2, \\ \dfrac{l_1 - u_2}{m_2 - u_2 + m_1 - l_1} & \text{otherwise.} \end{cases} \quad (5)$$

then the degree of possibility for a convex fuzzy number can be calculated by:

$$V(\tilde{a}_2 \geq \tilde{a}_1) = \text{hgt}(\tilde{a}_1 \cap \tilde{a}_2) = \frac{l_1 - u_2}{m_2 - u_2 + m_1 - l_1} = d \quad (6)$$

where d is the ordinate of the highest intersection point between $\tilde{a}_1$ and $\tilde{a}_2$ (see Figure 2) and the term hgt indicates the height of fuzzy numbers on the intersection of $\tilde{a}_1$ and $\tilde{a}_2$ (see [16]).
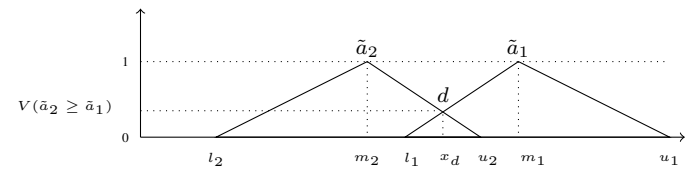

Figure 2. The degree of possibility for $\tilde{a}_2 \geq \tilde{a}_1$

Point x_d in Figure 2 indicates the point in the domain of $\tilde{a}_1$ and $\tilde{a}_2$ where the ordinate d is found [16]. Finally, we measure the weight vector for the criteria, assuming:

$$d'(A_i) = \min V(\tilde{S}_i \geq \tilde{S}_j), j = 1, 2, ..., n, j \neq i$$

where $A_i (i = 1, 2, ..., m)$ are the m decision alternatives and n is the number of criteria, then the weight vector is obtained by [16]:

$$W'(A_i) = (d'(A_1), d'(A_2), ..., d'(A_m))^T, A_i (i = 1, 2, ..., m) \quad (7)$$

By normalizing Eq. (7) we are able to compute the

normalized weight vectors [21]:

$$W(A_i) = (d(A_1), d(A_2), ..., d(A_n))^T \quad (8)$$

where W is a non-fuzzy number and represents the arrangement of the alternatives.

Also by dividing the normalized weight for every criteria on the sum of the normalized weight vectors, we are able to compute the importance degrees of the criteria:

$$W_{C_j} = \frac{W(A_j)}{\sum_{i=1}^{n} W(A_i)}, j = 1, ..., n \quad (9)$$

where W_{C_j} represents the importance degrees of a criterion and n is the number of criteria.

In the next section through an example, we use FAHP by the mentioned equations for solving a multi criteria decision making problem.

III. PROPOSED APPROACH

In this section, we describe our approach on how to use FAHP to prioritize test cases. AHP serves as a powerful tool in calculating weights to solve a multi-criteria decision making problem, but this method is not able to handle uncertainty in the decision problems and also ranking of AHP is partly imprecise [22]. To solve decision making problems which consist of uncertainty and vagueness, fuzzy sets provides a pairwise comparison as an extension which provides a more accurate description of the linguistic variables within the process of decision making [22].

We summarize the steps of our approach for solving a typical DSS problem in the following points:

1) Criteria Identification
2) Alternative Determination
3) Effect Measurement
4) Fuzzification
5) Apply AHP Technique for DSS

In the first step of the proposed approach, we need to identify different criteria which have direct effect on the alternatives, a typical decision making problem can be defined as single or multiple criteria.

In the second step, we define and analyse some possible solutions which are referred to the alternatives.

In the third step, we measure the effect of the criteria on the every single alternatives, to perform this part, we can use linguistic or numerical variables, which depends on the decision making situation.

In the fuzzification part, we interpret our measurement into a fuzzy set by using fuzzy rules and reasoning. Note that the fuzzification part only applies to the linguistic variables, if we have some numerical or sharp values for the effect measurement part, we jump to the last step, which is applying a decision making technique to solve the initial problem.

As last step, we suggest AHP technique for DSS, other decision making technique such as TOPSIS, which covers the fuzzy rules, can be applied in this part.

In our previous work [5], we assumed the cost-effort of the test case as the only criterion in the decision making process. The cost-effort estimation for a test case could, for example,

be the time, effort, and functional cost and budget that needs to be spent to perform each test.

In the present work, we assume a multiple-criteria decision making problem. The following set of test case properties are considered in this work as the main criteria for prioritization of test cases in the form of solving a DSS problem. The approach is not, however, limited to any particular set of test case properties as decision making criteria. In different systems and contexts users can have their own set of key test case properties based on which prioritization is performed.

- **Cost efficiency** (C_1) is used to capture the cost of a test case implementation, hardware setup cost, test case configuration cost (environment parameters), etc. The higher the cost efficiency degree of a test case, the more favorable it is.

- **Time efficiency** (C_2) is used to refer to a test case total execution time, test environment setup time and test case creation time. A test case with higher time efficiency is considered less time-costly.

- **Requirements coverage** (C_3) represents the number of requirements tested and covered by the test case.

- **Fault detection probability** (C_4) indicates the average probability of detecting a fault by each single test case.

- **Verdict Conclusiveness** (C_5) shows how conclusive and informative the verdict and result of a test case is. This is particularly interesting and more important for extra-functional aspects of a system where the meaning of a pass or fail result should be carefully investigated; for instance, a failure for user-friendliness or scalability [23].

- **Deviation Indicator** (C_6) is not per se and directly a property of a test case but that of a requirement. We have defined it in our previous work in [5] as the deviation degree of a requirement's satisfaction level from its ideal satisfaction level, i.e., when it is fully satisfied. It is calculated during the analysis of the requirements model. As it indicates which parts of a system can potentially have more severe problems with respect to the satisfaction of the requirements, we also include and use this property as one of the decision making criteria in the example case in this paper to also prioritize for test cases that target requirements with higher deviation indicator value.

IV. CASE SCENARIO

In this section, through an example, we show how it becomes possible to use the result of model-based analysis to guide testing efforts. Hence, test cases can be prioritized by applying Fuzzy AHP. The application of the non-functional requirements (NFR) profile in building and customizing a laptop computer product, has been simulated by Figure 3. There are several non-functional requirements that are defined for this system such as low boot-up time, increased battery life and security and to satisfy each, several features are used and applied.

For example, to satisfy the security requirement, having the option to use the BIOS password checking at start up time and also finger print mechanism for authentication are considered. However, the use of such features has also impacts on other parts of the system. For example, using a password check during
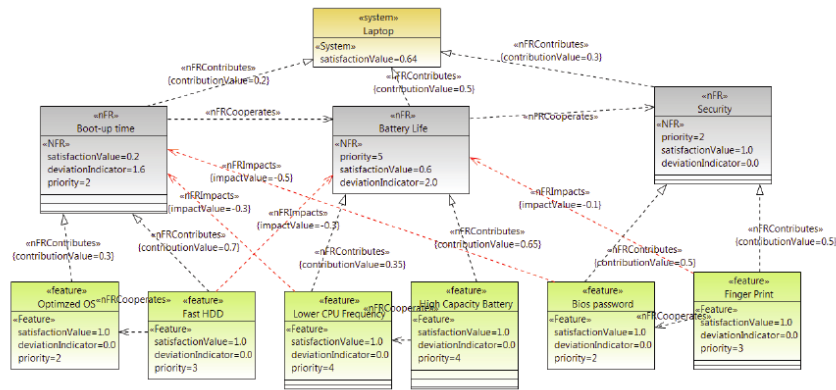
Figure 3. Analyzed model of the laptop system (Case Scenario)

the boot-up process affects the requirement to have low boot-up time negatively. Similarly, adding the finger print feature will add to the energy consumption of the system and thus affects the battery life time. These impacts and dependencies are established using the NFRImpacts links which are shown in red colour in Figure 3. The magnitudes of these impacts are stated on each of these links through the impactValue property. Customer preferences are captured by setting the priority properties. To test this system, there are 10 various test cases that target and cover its different requirements. As first step in the proposed approach, we need to identify the effective criteria and determine different alternatives.

Let $A = \{TC_1, TC_2, \ldots, TC_{10}\}$ be the set of test cases (alternatives) and $C = \{C_1, C_2, C_3, \ldots, C_6\}$ represents a set of the criteria that mentioned in proposed approach where $C_1 =$ Cost efficiency, $C_2 =$ Time efficiency, etc.

Figure 4 illustrates the relationships between the criteria and various test cases, as we see, the situation of decision making is symmetric where every criteria have a direct effect on every single test cases. The goal of this DSS problem is test cases prioritization. In the third step, we measure the effect of
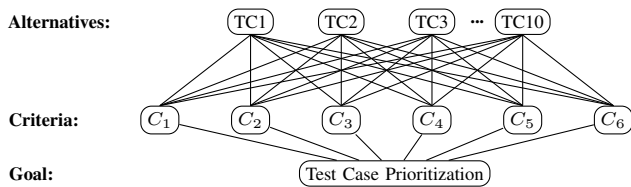

Figure 4. AHP hierarchy for prioritizing test cases

the criteria on the every single test cases. This effect has been assumed by the linguistic variables (e.g., low, high, etc.) and has been summarized in Table II.

TABLE II. THE PAIRWISE COMPARISON MATRIX FOR THE CRITERIA, WITH VALUES VERY LOW(VL), LOW (L), MEDIUM (M), HIGH (H) AND VERY HIGH (VH)

Test Case	Req. ID	C_1	C_2	C_3	C_4	C_5	C_6
TC1	RQ2	H	H	VH	H	VH	H
TC2	RQ1	M	M	H	M	M	VH
TC3	RQ1	M	H	H	H	VH	L
TC4	RQ3	VL	H	M	H	H	VH
TC5	RQ2	VH	M	M	VH	H	VH
TC6	RQ3	L	H	VH	H	VH	M
TC7	RQ1	M	L	L	VH	H	H
TC8	RQ3	VL	H	H	M	M	VH
TC9	RQ3	L	VH	VH	M	M	L
TC10	RQ2	VH	H	M	H	VH	M

The data in Table II are an empirical validation of the criteria effects. In the fuzzification phase (step 4), we interpret the effect of the various criteria on the test cases to a fuzzy set, to do these, Figure 1 and Table I have been used. As last step of the proposed approach, we apply AHP technique for prioritizing the test cases by using Eqs. (1) to (8).
The fuzzy pairwise comparison matrices for the alternatives and criteria becomes as follow by using Eq. (1):

$$
C_1 = \begin{array}{c} \\ A_1 \\ A_2 \\ \\ \vdots \\ A_{10} \end{array}
\begin{array}{cccc} A_1 & A_2 & \cdots & A_{10} \\ \left(\begin{matrix} \tilde{1} & \tilde{2} & \ldots & \tilde{2}^{-1} \\ \tilde{2}^{-1} & \tilde{1} & \ldots & \tilde{3} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{2} & \tilde{3}^{-1} & \ldots & \tilde{1} \end{matrix} \right) \end{array}
\quad
C_2 = \begin{array}{cccc} A_1 & A_2 & \cdots & A_{10} \\ \left(\begin{matrix} \tilde{1} & \tilde{2}^{-1} & \ldots & \tilde{5} \\ \tilde{2} & \tilde{1} & \ldots & \tilde{3} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{5}^{-1} & \tilde{3}^{-1} & \ldots & \tilde{1} \end{matrix} \right) \end{array}
$$

$$
C_3 = \begin{array}{cccc} A_1 & A_2 & \cdots & A_{10} \\ \left(\begin{matrix} \tilde{1} & \tilde{4} & \ldots & \tilde{1} \\ \tilde{4}^{-1} & \tilde{1} & \ldots & \tilde{1} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{1}^{-1} & \tilde{1}^{-1} & \ldots & \tilde{1} \end{matrix} \right) \end{array}
\quad
C_4 = \begin{array}{cccc} A_1 & A_2 & \cdots & A_{10} \\ \left(\begin{matrix} \tilde{1} & \tilde{6} & \ldots & \tilde{2} \\ \tilde{6}^{-1} & \tilde{1} & \ldots & \tilde{3} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{2}^{-1} & \tilde{3}^{-1} & \ldots & \tilde{1} \end{matrix} \right) \end{array}
$$

$$
C_5 = \begin{array}{cccc} A_1 & A_2 & \cdots & A_{10} \\ \left(\begin{matrix} \tilde{1} & \tilde{8} & \ldots & \tilde{1} \\ \tilde{8}^{-1} & \tilde{1} & \ldots & \tilde{2} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{6}^{-1} & \tilde{3}^{-1} & \ldots & \tilde{1} \end{matrix} \right) \end{array}
\quad
C_6 = \begin{array}{cccc} A_1 & A_2 & \cdots & A_{10} \\ \left(\begin{matrix} \tilde{1} & \tilde{5} & \ldots & \tilde{9} \\ \tilde{6}^{-1} & \tilde{4}^{-1} & \ldots & \tilde{7} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{9}^{-1} & \tilde{8}^{-1} & \ldots & \tilde{1} \end{matrix} \right) \end{array}
$$

TFNs are calculated by using Eq. (3) as explained in the background section:

$$
\sum_{i=1}^{n} \sum_{j=1}^{m} \tilde{a}_{ij} = (43.1, 72.4, 98.5)
$$

and also Eq. (4) gives us the inverse of numbers:

$$
\left[\sum_{i=1}^{n} \sum_{j=1}^{m} \tilde{a}_{ij} \right]^{-1} = (0.023, 0.013, 0.010)
$$

the value of fuzzy synthetic extent $\tilde{S}_i$ can be obtained by using Eq. (2):

$$
S_1 = (0.01, 0.09, 0.11), S_2 = (0.02, 0.0.24, 0.21)
$$
$$
S_3 = (0.01, 0.02, 0.03), S_4 = (0.06, 0.23, 0.24)
$$
$$
S_5 = (0.03, 0.33, 0.36), S_6 = (0.01, 0.15, 0.18)
$$

we determine the weight rating for each criteria by using Eq. (5),

define $V_{ij} = V(\tilde{S}_i \geq \tilde{S}_j)$ then:

$V_{12} = 1, V_{13} = 1, V_{14} = 1, V_{15} = 1, V_{16} = 1$

$V_{21} = 0.67, V_{23} = 1, V_{24} = 1, V_{25} = 1, V_{26} = 1$

$\vdots$

$V_{61} = 0.66, V_{62} = 0.48, V_{63} = 0.20, V_{64} = 0.90, V_{65} = 0.07$

In this step, we compute the normalized and unnormalized weights for the criteria by using Eqs. (7), (8). The results have been summarized in Table III:

TABLE III. THE WEIGHT OF THE CRITERIA

Criteria	Unnormalized Weight	Normalized weight
C_1	0.58	0.02
C_2	0.67	0.25
C_3	0.28	0.01
C_4	0.97	0.36
C_5	0.02	0.22
C_6	0.07	0.10

In the next step we compare the weights for every single criteria with the alternatives, to avoid lengthy calculations we summarize the result in Table IV.

TABLE IV. COMPARISON THE WEIGHTS OF ALTERNATIVES WITH CRITERIA

Criteria	C_1	C_2	C_3	C_4	C_5	C_6
Weight	0.02	0.25	0.01	0.36	0.22	0.10
A_1	0.333	0.096	0.460	0.520	0.595	0.409
A_2	0.123	0.010	0.121	0.012	0.009	0.203
A_3	0.201	0.014	0.198	0.258	0.239	0.257
A_4	0.224	0.023	0.332	0.402	0.321	0.298
A_5	0.233	0.025	0.351	0.430	0.475	0.319
A_6	0.205	0.017	0.207	0.261	0.237	0.252
A_7	0.221	0.020	0.210	0.298	0.245	0.264
A_8	0.144	0.012	0.132	0.022	0.111	0.218
A_9	0.104	0.003	0.110	0.008	0.005	0.098
A_{10}	0.226	0.015	0.232	0.309	0.311	0.248

By normalizing the values in Table IV, we are able to prioritize the test cases, the result has been illustrated in Figure 5:
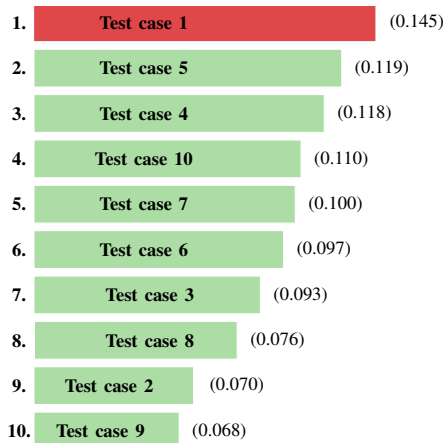

Figure 5. Test cases prioritization result

As can be seen, test case number 1 has the highest weight 0.145 among the test cases and it tests the requirement number 2. The proposed DSS approach prioritizes the test cases as the following set:

$\{TC_1, TC_5, TC_4, TC_{10}, TC_7, TC_6, TC_3, TC_8, TC_2, TC_9\}.$

Moreover, via Eq. (9), the importance degrees of the criteria are computed and summarized in Table V.

TABLE V. CRITERIA IMPORTANCE

Criteria	Importance
1. Fault Detection	0.37
2. Time Efficiency	0.26
3. Verdict Conclusiveness	0.22
4. Deviation Indicator	0.10
5. Cost Efficiency	0.02
6. Requirement Coverage	0.01

As can be seen, fault detection is identified as the most critical decision criterion which has the highest weight 0.37 among all the criteria.

As mentioned earlier, the most critical criterion is the one for which the smallest change in its current weight will alter the existing ranking of the alternatives [24].

V. CONCLUSION AND FUTURE WORK

In this paper, we introduced a test case prioritization approach based on the fuzzy AHP decision making technique. Our approach enables to rank test cases based on a set of criteria and is particularly necessary when there is a limitation (due to resource constraints) in selection and execution of test cases for a system and it is not possible to run all the test cases. We demonstrated how the approach is applied using an example set of test case properties including cost efficiency, time efficiency, verdict conclusiveness, etc., that serve as criteria in the prioritization process. The approach is not, however, limited to this particular set of criteria. The extension and use of AHP in a fuzzy environment allows to specify the degree of a criterion in each alternative (i.e., a test case) using linguistic variables which relaxes the need of the users of the approach to specify precise quantified values. This can improve the practicality and adoption of the approach; for instance, where only estimated and imprecise data are available. As part of the decision making process, it was also identified which of the chosen criteria has a higher role in determining the ranking of the test cases. Use of this information on importance degree of each criteria in other analyses of test cases would be another interesting topic to further investigate.

As a future work, we are also going to examine other multi-criteria decision making techniques in prioritization of test cases. As another research direction of this work, finding a solution to apply a test case prioritization approach as part of the test case generation process would be an ideal scenario. Having such a solution would then enable to only generate a set of test cases which are analysed and deemed as feasible to execute with respect to the available resources for test execution. As for other works on test case prioritization techniques in the literature, generally fault detection rate is mainly used as the single important criterion in prioritizing and ranking of test cases. In this paper, we have considered test case prioritization in a broader aspect covering various and multiple criteria. Also in the application of the proposed approach, same priority level and importance was considered for all the criteria. If in a different context, one or more criteria are considered more important than the others (e.g., fault detection), a higher importance degree can be assigned to them during the pair-wise comparison process of the criteria. Considering the dependencies that can exist between test cases is also another extension of this work which we are currently investigating. Such dependency information

can as well be used in prioritization and ordering the execution of test cases. Therefore, there is the potential to combine and include it as another criterion in the decision making solution we proposed in this paper or use it in a separate step before or after the prioritization based on the other criteria.

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References

[1] S. Yoo and M. Harman, "Pareto efficient multi-objective test case selection," in Proceedings of the 2007 International Symposium on Software Testing and Analysis, ser. ISSTA '07. New York, NY, USA: ACM, 2007, pp. 140–150.

[2] H. Do, S. Mirarab, L. Tahvildari, and G. Rothermel, "The effects of time constraints on test case prioritization: A series of controlled experiments," Software Engineering, IEEE Transactions on, vol. 36, no. 5, Sept 2010, pp. 593–617.

[3] A. Z. Dario Di Nucci, Annibale Panichella and A. D. Lucia, "Hypervolume-based search for test case prioritization," in Proceedings of the 2015 Symposium on Search-Based Software Engineering (SSBSE'15). Bergamo, Italy: Springer, 2015, pp. 140–150.

[4] G. Rothermel, R. Untch, C. Chu, and M. Harrold, "Test case prioritization: an empirical study," in Software Maintenance, 1999. (ICSM '99) Proceedings. IEEE International Conference on, 1999, pp. 179–188.

[5] M. Saadatmand and M. Sjödin, "On combining model-based analysis and testing," in Information Technology:New Generations (ITNG), 10th International Conference on, Las Vegas, USA, April 2013, pp. 260–266.

[6] M. Harman, "Making the case for morto: Multi objective regression test optimization," in Software Testing, Verification and Validation Workshops (ICSTW), 2011 IEEE Fourth International Conference on, March 2011, pp. 111–114.

[7] T. L. Saaty, "How to make a decision:the analytic hierarchy process," European Journal of Operational Research, vol. 48, no. 1, 1990, pp. 9–26.

[8] R. Bellman and L. Zadeh., "Decision making in a fuzzy environment," Management Science, 1970, pp. 141–164.

[9] H. Jantan, A.Hamdan, and Z. Othman, "Intelligent techniques for decision support system in human resource management." 2010.

[10] V. Novák, I. Perfilieva, and J. Mockor, Mathematical principles of fuzzy logic. Springer Science & Business Media, 2012, vol. 517.

[11] D. Rabunal Dopico and Pazos, Encyclopedia of Artificial Intelligence. University of A Coruna, Spain, 2009, vol. 3.

[12] Y. Shi, "A Deep Study of Fuzzy Implications," Ph.D. dissertation, Ghent University, 2009.

[13] L. Zadeh, "Fuzzy sets," Information and Control, 1965, pp. 338–353.

[14] J. Yang and J. Watada, "Fuzzy clustering analysis of data mining: Application to an accident mining system," International Journal of Innovative Computing, Information and Control, 2012, pp. 5715–5724.

[15] E. E. K. B. De Baets, "Fuzzy relational compositions," Fuzzy Sets and Systems, vol. 60, no. 1, 1993, pp. 109 – 120.

[16] Y.-C. Tang, "Application of the fuzzy analytic hierarchy process to the lead-free equipment selection decision," Business and Systems Research, 2011, pp. 35–56.

[17] J. Malczewski, Multiple Criteria Decision Analysis and Geographic Information Systems, M. Ehrgott, J. R. Figueira, and S. Greco, Eds. Springer US, 2010, vol. 142.

[18] C. Carlsson and R. Fuller, "Fuzzy multiple criteria decision making: Recent developments," Fuzzy Sets and Systems, vol. 2, no. 1, June 1996, pp. 415–437.

[19] K. Shahroodi, S. Amini, E. Shiri, K. S. Haghighi, and M. Najibzadeh, "Application of analytical hierarchy process (ahp) technique to evaluate and selecting suppliers in an effective supply chain," Arabian Journal of Business and Management Review, vol. 1, no. 8, April 2012.

[20] T. Terano, Fuzzy Engineering Toward Human Friendly Systems. Ohmsha, 1992, no. v. 2.

[21] D.-Y. Chang, "Theory and methodology applications of the extent analysis method on fuzzy ahp," European Journal of Operational Research, 1996, pp. 649–655.

[22] G. Kabir and M. A. A. Hasin, "Comparative Analysis of AHP and Fuzzy AHP Models for Multicriteria Inventory Classification," International Journal of Fuzzy Logic Systems, vol. 1, no. 1, 2011, pp. 87 – 96.

[23] R. M. Hierons, "Verdict functions in testing with a fault domain or test hypotheses," ACM Trans. Softw. Eng. Methodol., vol. 18, no. 4, Jul. 2009, pp. 14:1–14:19.

[24] E. Triantaphyllou, B. Kovalerchuk, L. Mann, and G. M. Knapp, "Determining the most important criteria in maintenance decision making," Journal of Quality in Maintenance Engineering, vol. 3, no. 1, 1997, pp. 16–28.